Extended Abstracts

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On nilpotency in braces and the Yang–Baxter equation

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Resum *(CAT)*
Les brides són estructures algebraiques que permeten estudiar les solucions no degenerades de l'equació de Yang–Baxter (EYB). Cada brida admet una solució no degenerada i, recíprocament, tota solució d'aquest tipus està determinada per una brida associada. Així, la classificació de les solucions no degenerades depén de l'anàlisi estructural de les brides. Les seues propietats algebraiques es corresponen amb les de les solucions, i la nilpotència permet descriure el caràcter multipermutacional d'aquestes estructures.

Keywords: *braces, Yang–Baxter, nilpotency.*

Abstract

The Yang–Baxter equation (YBE) is a fundamental equation in theoretical physics, arising independently in the works of C. N. Yang (1967), Nobel Laureate in Physics, and R. J. Baxter, within the frameworks of quantum field theory and the study of integrable models in statistical mechanics, respectively.

The formulation of the YBE is strongly inspired by the celebrated Reidemeister moves (cf. [3]).

Consequently, the study of YBE solutions has gained significant relevance in recent decades, both because of its intrinsic importance and its applications in braid theory, braided groups, quantum groups, cryptography, and noncommutative geometry.

The multidisciplinary context of the YBE has generated great interest in the search for and classification of its solutions.

Open Problem. To find and classify the solutions of the Yang–Baxter equation.

Given the Herculean nature of this task, the Fields Medalist V. G. Drinfeld ([2]) proposed focusing on the so-called set-theoretic solutions of the YBE, a type of combinatorial solution whose geometric and symmetric character naturally gives rise to algebraic techniques.

In this work, we undertake a thorough analysis of the algebraic property of nilpotency in braces, as a clear and significant example of the translation of algebraic properties into classificatory properties of YBE solutions. We study the so-called lateral nilpotencies in braces, which have a distinct impact both on the structural analysis of braces and on the classification of solutions. In this context, a key concept of nilpotency in braces—one that has recently emerged and has a decisive impact both structurally within braces and classificatorily within YBE solutions—is central nilpotency in braces. This type of nilpotency arises with the aim of unifying both lateral nilpotencies in braces, as shown in [1], where it is demonstrated that central nilpotency in braces can be regarded as the true analogue, within brace theory, of group nilpotency.

Within group theory, the local study of nilpotency or p-nilpotency associated with a prime p has undergone substantial development following the seminal works of Hall and Higman (cf. [4]). A key concept in this context is the p-Fitting subgroup of a finite group, the largest normal p-nilpotent subgroup of the group.

The main objective and contribution of this work is the introduction and analysis of central p-nilpotency in finite braces. We conduct a comprehensive structural study of central p-nilpotency in braces, allowing us to define an appropriate p-Fitting ideal. This contribution is original within the theory and is intended to inspire further developments in this field.

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