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Extension of ϕ-Lipschitz functions

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Resum *(CAT)*
Els teoremes d'extensió de McShane i Whitney poden generalitzar-se de diverses maneres, permetent estendre funcions des de subespais metrics a l'espai sencer preservant la constant de Lipschitz. En aquest sentit, proposem introduir una funció ϕ creixent, positiva i subaditiva que, en compondre-la amb una mètrica, s'obté una altra mètrica que millora l'extensió. A més, la noció d'espai d'índexs és introduïda i els resultats d'aquest treball generalitzen els ja coneguts sobre índexs de Lipschitz per al cas dels ϕ-Lipschitz.

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Abstract

The classical Lipschitz extension theorems for real functions, due to McShane and Whitney, have found numerous applications in various fields: economics, social sciences and, more recently, in the field of artificial intelligence. These results can be generalised in various ways, by extending the class of functions to which they can be applied, or by weakening the metric conditions. In all these cases, we can extend functions defined on metric subspaces to the simple space, preserving the Lipschitz constant. In this sense, the proposal of this work consists in introducing an increasing, positive and subadditive function ϕ which, when composed with a metric, obtains another function with similar properties to the original metric.

Furthermore, in order to provide a functional basis for the recent interest in numerical indices in various disciplines (stock markets, forecasting, demography, etc.), the notion of index space is introduced. These indices are real Lipschitz functions which, depending on the problem, may satisfy additional conditions such as the Katetov condition. The results of this work generalise the already known results on Lipschitz indices for the case of ϕ-Lipschitz, in addition to studying the compactness of the set of corresponding standard indices. The properties of the approximation that make it possible to work with this functional basis to design artificial intelligence algorithms in ϕ metric models are also presented.

Finally, this paper presents and contextualises a problem of recent interest related to urban liveability indices. We will see how modelling through index expansions and their respective extensions will be useful, and will serve as an example to compare the methodology we have developed with the original one. The results obtained show that, in general, the extension process is improved by composing the original metric with a ϕ function, on the nature of which the final result will depend. For further reference see [1, 2, 3, 4].

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