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Lasing without population inversion: towards X-ray continuous wave laser emission

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Abstract

This paper is devoted to lasing without population inversion in three and multi-level atomic configurations in interaction with a driving laser. First we discuss the interest and the current state-of-the-art of this phenomenon. Then, the basic physics of lasing without inversion is described in more detail and analyzed in terms of quantum interference between the dressed-states built up by the driving field. In addition, the quantum-jump formalism is used to introduce generalized Einstein *B* coefficients accounting for stimulated emission and absorption processes in these schemes. As a result, lasing without inversion is explained in terms of non-reciprocity between stimulated emission and absorption coefficients. Likewise, this formalism allows us to analyze lasing without inversion in terms of the quantum Zeno effect.

Some particular problems of lasers without inversion are also investigated, such as, laser emission with a driving field of frequency smaller than the one we want to generate without population inversion; lasing without inversion in the absence of an external driving field; and electromagnetically induced transparency as a first step to amplification without inversion.

Finally, some dynamical features of these lasers are explored such as the inversionless laser emission in a self-pulsing regime or the generation of giant pulses of laser light.

Resum

Aquest article de revisió se centra en el fenomen de l'emissió làser sense inversió de població en configuracions atòmiques de tres o més nivells en interacció amb un làser de preparació. En primer lloc, es descriu breument l'interès i l'estat actual de la recerca al voltant d'aquest fenomen. A continuació, es descriu amb detall el fonament físic de l'emissió làser sense inversió i així s'analitza aquest fenomen en termes d'interferència quàntica entre els estats atòmics vestits pel làser de preparació. Paral·lelament, s'utilitza el formalisme dels salts quàntics per introduir coeficients *B* d'Einstein generalitzats per a l'emissió estimulada i l'absorció en aquests esquemes i com a consequència, l'emissió làser sense inversió de població s'explica en termes d'una no-reciprocitat en els coeficients per a l'emissió estimulada i l'absorció. Així mateix, aquest formalisme permet analitzar l'emissió làser sense inversió en termes de l'efecte Zenó quàntic.

A més a més, s'investiguen alguns problemes concrets dels làsers sense inversió de població com, per exemple i l'emissió làser sense inversió amb un camp làser de preparació de freqüència menor que la del generat sense inversió, l'emissió làser sense inversió en absència d'un camp de preparació extern i la transparència induïda electromagnèticament com a pas previ per a l'amplificació sense inversió.

Finalment, s'exploren alguns aspectes particulars de la dinàmica d'aquests làsers com són l'emissió làser sense inversió en règim autopolsant o la generació de polsos gegants de llum làser.

Keywords: Atomic coherence effects, amplification without inversion (AWI), lasing without inversion (LWI), inversion without lasing (IWL), electromagnetically induced transparency (EIT), quantum Zeno effect.

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Over the last few years, atomic coherence effects such as coherent population trapping (CPT), electromagnetically induced transparency (EIT), high index of refraction with vanishing absorption, amplification and lasing without population inversion (AWI and LWI) and population inversion without amplification or lasing (IWA and IWL) have been intensively investigated due to their multiple potential applications. These applications range from laser cooling to isotope separation and from ultrahigh-sensitive magnetometers to the generation of giant pulses of laser light [1-4]. Among all these phenomena, AWI and LWI have received considerable attention for their potential to obtain laser light in spectral domains, e.g., the x-ray domain, where conventional methods based on population inversion are not available or difficult to implement.

In this paper we present LWI and indicate the current state-of-the LWI experiments. Then, the basic physics of this phenomenon will be discussed in terms of quantum interference between the dressed-states built up by the external driving field and by making use of the quantum-jump formalism. Finally, we explore some particular problems in the way towards x-ray lasing and some dynamical features of these lasers.

Why lasing without population inversion?

At the beginning of the century, the combination of Bohr's atomic model with a stochastic conception of the light-matter interaction allowed Einstein to establish the existence of three basic processes in the interaction of light with matter: absorption, stimulated emission, and spontaneous emission. In the context of black-body radiation and from thermodynamical arguments, Einstein determined –in terms referred to as Einstein A and B coefficients– the relationship between the rates of absorption and stimulated emission of light by a gas molecule possessing discrete energy levels and the spontaneous emission rate –given by the Einstein A coefficient. For a two-level system with ground level $|g\rangle$ and excited level $|e\rangle$ with unperturbed transition frequency ω probed by an electromagnetic field, the relationships between the Einstein coefficients read

$$B_{abs} = B_{st\ em} (\equiv B) \tag{1a}$$

$$\frac{A}{B} = \frac{\hbar\omega^3}{\pi^2 c^3} \tag{1b}$$

with

$$A = \frac{\omega^3}{3\pi\varepsilon_0 \hbar c^3} |\mu|^2 \tag{2}$$

where μ is the electric dipole moment of the two-level transition, $\hbar$ Planck's constant, ε_0 the electric permittivity, and c the speed of light in vacuum. Thus, the rates of absorption and stimulated emission of light by an atomic/molecular medium will be proportional to the corresponding Einstein B coefficient times the population ρ_{ii}($i=g,e$) of the initial state of the process. Then,

$$\frac{\text{absorption rate}}{\text{stimulated emission rate}} = \frac{B_{abs}}{B_{st\ em}} \cdot \frac{\rho_{gg}}{\rho_{ee}} \tag{3}$$

From the above equations, it can be deduced straightforwardly that (1) since $B_{abs}=B_{st\ em}$ then population inversion, i.e., $\rho_{ee}>\rho_{gg}$, is a necessary condition for light amplification and (2) it is not possible to invert a two-level system with, for instance, resonant light, since atomic excitation is always accompanied with de-excitation through the stimulated emission process. As a consequence, in order to create the required population inversion, laser systems operate on three- or more level configurations (e.g., the three-level configuration of the Ruby laser or the four-level configuration of the He-Ne laser), where the inversion is created by pumping to an excited level which rapidly decays to the upper level of the lasing transition.

In these schemes, in order to reach laser action the incoherent pump power P should be strong enough to create a population inversion such that the associated gain overcomes cavity losses. A simple estimation of the lower bound P_{th} for the pump power required to reach laser oscillation gives [5]:

$$P_{th} = \frac{\hbar\omega\Delta N_{th}}{\tau} \tag{4}$$

where $\Delta N_{th}>0$ is the population difference at the first laser threshold and τ the lifetime of the upper level of the lasing transition. This threshold population can be written [6] as $\Delta N_{th}=2\kappa/\hbar\omega B(\omega)$, κ being the cavity losses and $B(\omega)$ the Einstein B coefficient at frequency ω. For a broadened atomic system with normalized spectral line given by $g(\omega)$ such that $B(\omega)=Bg(\omega)$ the above expression becomes:

$$P_{th} = \frac{2\kappa}{g(\omega)} \frac{A}{B} = \frac{2\kappa\hbar\omega^3}{\pi^2 c^3 g(\omega)} \tag{5}$$

where use of relation (1b) has been made. In general, and due to the normalization of $g(\omega)$, one has $g(\omega)\cdot\Delta\omega\sim 1$. On the other hand, for natural broadening and using the Heisenberg principle one has $\Delta\omega\sim 1/\tau$ with $1/\tau=A$, while for Doppler broadening it is well known that $\Delta\omega$ scales with ω and does not depend on the electric dipole moment [5]. Therefore, according to equation (5) the threshold pump power scales with ω^6 and $|\mu|^2$ for natural broadening, while for Doppler broadening it scales with ω^4 and does not depend on the dipole moment [5]. All these arguments clearly show that as we increase the frequency of the laser transition it becomes harder to attain the required population inversion. Therefore, in continuous-wave x-ray lasing, the main obstacle for the achievement of coherent oscillation is the required pump power

LWI differs from conventional lasing in that the reciprocity between absorption and stimulated emission is broken. In LWI the absorption of light is reduced or even cancelled, and lasing is possible even with a small fraction of the atoms in the upper level (less than in the lower level) of the lasing transition. Therefore, LWI is not subject to the limitations of conventional lasing, i.e., the required incoherent pump power can be drastically reduced, and, in this sense, LWI opens a new way towards the generation of continuous-wave short-wavelength lasers.

State-of-the-art of lasing without inversion

As early as 1957, Javan showed, in the context of the theory of a three-level maser, the possibility of inversionless amplification in the so-called *V*-type three-level system [7]. In Javan's scheme, an intense maser field saturated the transition of one of the arms of a *V* scheme while another microwave coherent field of smaller frequency probed the adjacent transition in the other arm. As pointed out by Javan: «...the lineshape at the amplifying frequency of the saturating power as derived in this paper, at first sight seem to contradict the results obtained from a treatment based on the population differences alone». In fact, it was shown by Javan that one can obtain a net induced emission at some portions of a resonant line and a net absorption at other frequencies within the line width due to the contribution of an interference term arising from the overlap between the Autler-Townes doublet built up by the driving field.

Unfortunately, LWI in coherently driven discrete three-level systems was not further investigated until three decades later. The seminal papers of Kocharovskaya and Khanin [8] as well as Scully and co-workers [9] marked the starting point of the current research on LWI. Up to now, several experiments have shown transient and continuous wave inversionless amplification [10-18] although only a few have reported laser oscillation without inversion [19-21]. In some cases, performing a basis transformation, LWI can be understood as arising from a hidden inversion as in the experiments reported in [20,11]. However, as in the first experimental demonstration of LWI [19], lasing without hidden inversion with respect to any meaningful basis transformation is also possible, and will be discussed in more detail below.

Basic physics of lasing without inversion

Quantum interference between dressed-state resonances

Let us consider the *V* and cascade schemes of figures 1(a) and 1(d), respectively. In both cases a resonant coherent field drives transition $|b\rangle-|c\rangle$ while a weak field probes the adjacent transition $|a\rangle-|b\rangle$. The coupling between the driving (probe) field and the $|b\rangle-|c\rangle$ $(|a\rangle-|b\rangle)$ transition is characterized by the Rabi frequency Ω_d (Ω_p). In the dressed state picture (Figures 1(b) and 1(e)), the driving field splits state $|b\rangle$ in two dressed states $|+\rangle$ and $|-\rangle$ with opposite detunings $\Delta_+(=-\Delta_-)=\Omega_d/2$ from state $|b\rangle$. Let us take a strong enough driving field such that it saturates transition $|b\rangle-|c\rangle$, i.e., $\rho_{cc}\approx\rho_{bb}$, and therefore, $\rho_{++}=\rho_{--}=\rho_{bb}=\rho_{cc}$. In this case, population inversion in the probed transition of the bare atomic system implies population inversion in the dressed-state basis and vice versa. Therefore, one should expect no probe gain for $\rho_{bb}>\rho_{aa}$. Surprisingly, a simple standard density-matrix calculation shows that even in this case probe amplification is possible, as illustrated in figures 1(c) and 1(f) by the probe response. For the parameter values used in these figures the dressed-states resonances contribute to probe field absorption since $\rho_{aa}<\rho_{++}=\rho_{--}$, even though inversionless amplification occurs between the dressed-state resonances for the *V* scheme and outside the dressed-state resonances for the cascade scheme.

Agarwal [23] analyzed the origin of AWI in both the bare and the dressed state bases in the Λ-type three-level system proposed by Imamoglu *et al.* [24], showing that gain arises from atomic coherence between the dressed states. In fact, due to this coherence the probe response corresponds approximately to two independent Lorentzian contributions at the dressed-states position plus an interference term consisting in two dispersive-like structures also located at the dressed-state position. Thus, figures 2(a) and 2(b) show the same probe response plotted in figures 1(c) and 1(f), but they are now split into two separate contributions: the inde-

Table 1. Experiments on pulsed and continuous wave (CW) AWI and LWI, and lasing below threshold inversion (LBT). *R* is the probe(lasing)-to-driving frequency ratio

Category	*Authors*	*Medium*	*Driving (nm)*	*Probe (lasing) (nm)*	*R*
Pulsed AWI	Nottelmann *et al.* (1993) [10]	Sm vapor cell (Λ)	570.68	570.68	1
Pulsed AWI	Fry *et al.* (1993) [11]	Na vapor cell (Λ)	589.86	589.86	1
			558.43	558.43	1
Pulsed AWI	van der Veer *et al.* (1993) [12]	Cd vapor cell (Λ)	326	479	0.68
CW AWI	Kleinfeld and Streater (1994, 1996) [13]	K vapor cell (4-level)	766.5	769.9	1
CW AWI	Zhu *et al.* (1996) [14]	Rb vapor cell (Λ)	780	780	1
CW AWI	Sellin *et al.* (1996) [15]	Ba atomic beam (cascade)	554	821	0.67
CW AWI	Fort *et al.* (1997) [16]	Cs vapor cell (V)	852	894	0.95
CW AWI	Shiokawa *et al.* (1997) [17]	Laser-cooled Rb atoms (Λ)	780	780	1
CW AWI	Hollberg *et al.* (1998, 1999) [18]	Laser-cooled Rb atoms (V)	780	795	0.98
CW AWI	Zibrov *et al.* (1995) [19]	Rb vapor cell (V)	780	795	0.98
CW LWI	Padmabandu *et al.* (1996) [20]	Na atomic beam (Λ)	589.76	589.43	1
Pulsed LWI	de Jong *et al.* (1998) [21]	Cd vapor cell (Λ)	326	479	0.68
CW LBT	Peters and Lange (1996) [22]	Ne vapor cell (double-Λ)	824.9	611.8	1.35

pendent dressed-states resonances (term A_1) and the interference between the dressed-states resonances (term A_2). In both cases, term A_1 contributes to probe absorption since there is no inversion between the upper level of the probed transition and the dressed states, i.e., $\rho_{aa}<\rho_{++}=\rho_{--}$. For the V scheme the interference term is destructive ($A_1A_2<0$) between the dressed-states resonances which gives rise to inversionless gain at line center. For the cascade scheme, the interference term is constructive ($A_1A_2>0$) between the dressed-states resonances and destructive outside this region which gives rise to inversionless gain at the wings of the dressed states resonances. A detailed analysis of the character of this interference term in folded and cascade schemes has been performed very recently by our group [25].

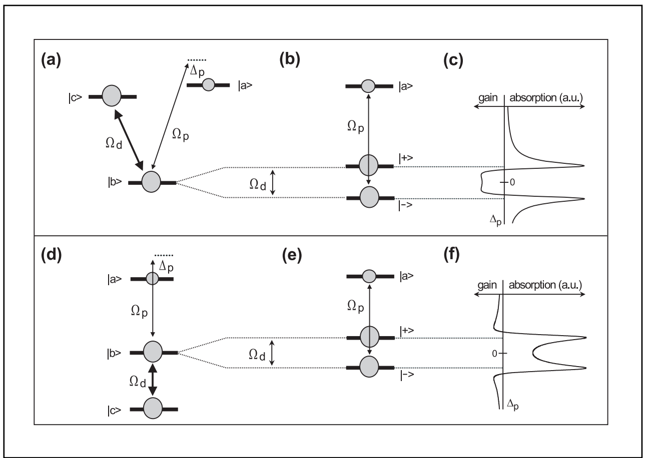

Figure 1. (a) Resonantly driven V-scheme without inversion in the bare atomic basis. (b) The same V-scheme without inversion in the dressed-state basis. (c) Probe response showing gain between the dressed-state resonances. (d) Resonantly driven cascade scheme without inversion in the bare atomic basis. (e) The same cascade scheme without inversion in the dressed-state basis. (f) Probe response showing gain outside the dressed-state resonances.

Quantum-jump approach to inversionless lasing

The quantum-trajectory or quantum-jump formalism [26-29] has been used in order to calculate the individual contribution of the different physical processes (one-photon and two-photon gain/loss processes) responsible for inversionless amplification. This formalism gives the same results as the standard density-matrix formalism but provides new insights into the underlying physical mechanisms [26,27]. In this formalism, the time evolution of the atom plus lasers system is pictured as consisting of a series of coherent evolution periods separated by quantum-jumps occurring at random times, i.e., a so-called quantum trajectory. The quantum-jumps are determined by dissipative processes, such as spontaneous emission or incoherent pumping, while the continuous evolution is governed by the coherent laser fields. Thus, a one-photon gain (loss) process is a coherent evolution period between two consecutive quantum-jumps such that the probe field photon number increases (decreases) by one with no change in the driving field photon number. A two-photon gain process corresponds to a coherent evolution period for which the probe field photon number increases by one and the driving field photon number decreases by one (V and Λ schemes) or in the cascade schemes, increases by one. Using this technique, Arimondo

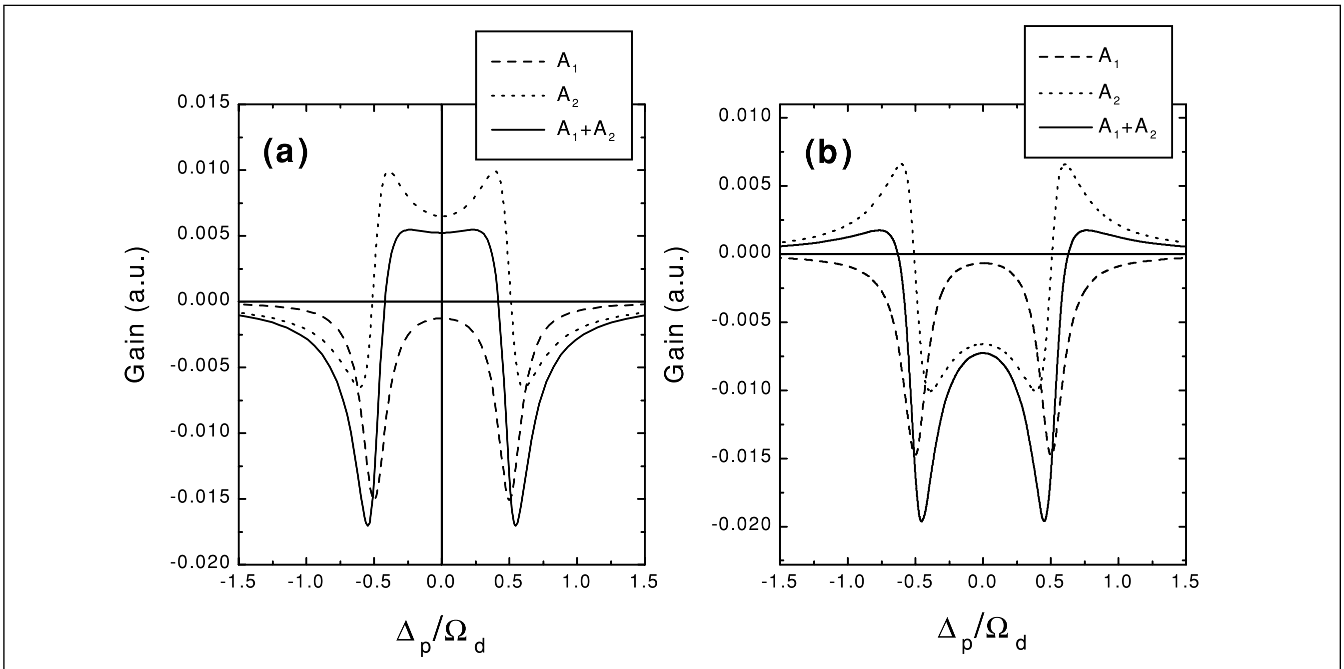

Figure 2. (a) Gain for a weak probe field as a function of its detuning (solid curve) with an on-resonance driving field for (a) a V-scheme and (b) a cascade scheme. The total gain profile (solid curve) is split into the independent contribution of the two-dressed states resonances, A_1 (dashed curve), and the contribution of the quantum interference between these resonances, A_2 (dotted curve). The parameters are: (a) $\gamma_{ab}=0.1\gamma_{cb}$, $\Lambda=\gamma_{cb}$, $\Omega_d=10\gamma_{cb}$ and $\Omega_p=0.002\gamma_{cb}$; and (b) $\gamma_{ab}=0.1\gamma_{bc}$, $\Lambda=\gamma_{bc}$, $\Omega_d=10\gamma_{bc}$, and $\Omega_p=0.002\gamma_{bc}$. γ_{ij} is the spontaneous population decay rate from $|i\rangle$ to $|j\rangle$ and Λ is the rate of a bi-directional incoherent pumping process acting on the probed transition.

[28] and Cohen-Tannoudji *et. al.* [29] revealed that AWI in folded schemes results from the fact that, for appropriate parameter values, two-photon gain processes overcome one-photon and two-photon loss processes even without one-photon or two-photon inversion. By contrast the quantum-jump formalism shows that the one-photon gain is the physical process responsible for inversionless gain in cascade schemes [25].

Through a quantum-jump analysis, Figures 3(a) and 3(b) show the individual contribution of the different physical processes to the total probe absorption spectrum corresponding to the *V* and cascade schemes of Figures 1 and 2. For the *V*-scheme configuration, two-photon gain processes predominate at line center and therefore they are the physical processes responsible for probe gain. Inversionless gain at the wings of the dressed-states resonances in the cascade scheme is due to the predominance in this region of one-photon gain processes. One of the benefits of the quantum-jump formalism relies on the fact that the knowledge of the particular physical processes responsible for inversionless gain allows one to select appropriate probe and driving field detunings to stimulate these processes while at the same time preventing the unfavorable ones. Thus, for the *V*-scheme being investigated one should detune both fields out from their respective atomic resonances but still fulfill the two-photon resonance condition in order to enhance the probability of two-photon processes while reducing the probability of one-photon processes. For the cascade scheme, one should detune the driving field out from its atomic resonance while maintaining the probe field tuned close to one-photon resonance.

Generalized Einstein B coefficients

As discussed above, the population inversion requirement for conventional lasers to operate is a direct consequence of the symmetry between the Einstein *B* coefficients for one-photon processes, in two-level lasers, and for two-photon processes, in Raman lasers. The quantum-jump formalism has been used very recently to define generalized Einstein *B* coefficients for one- and two-photon gain and loss processes [30]. Thus, it has been shown that in three-level systems coherently driven close to resonance there is a symmetry breaking between the Einstein *B* coefficients for

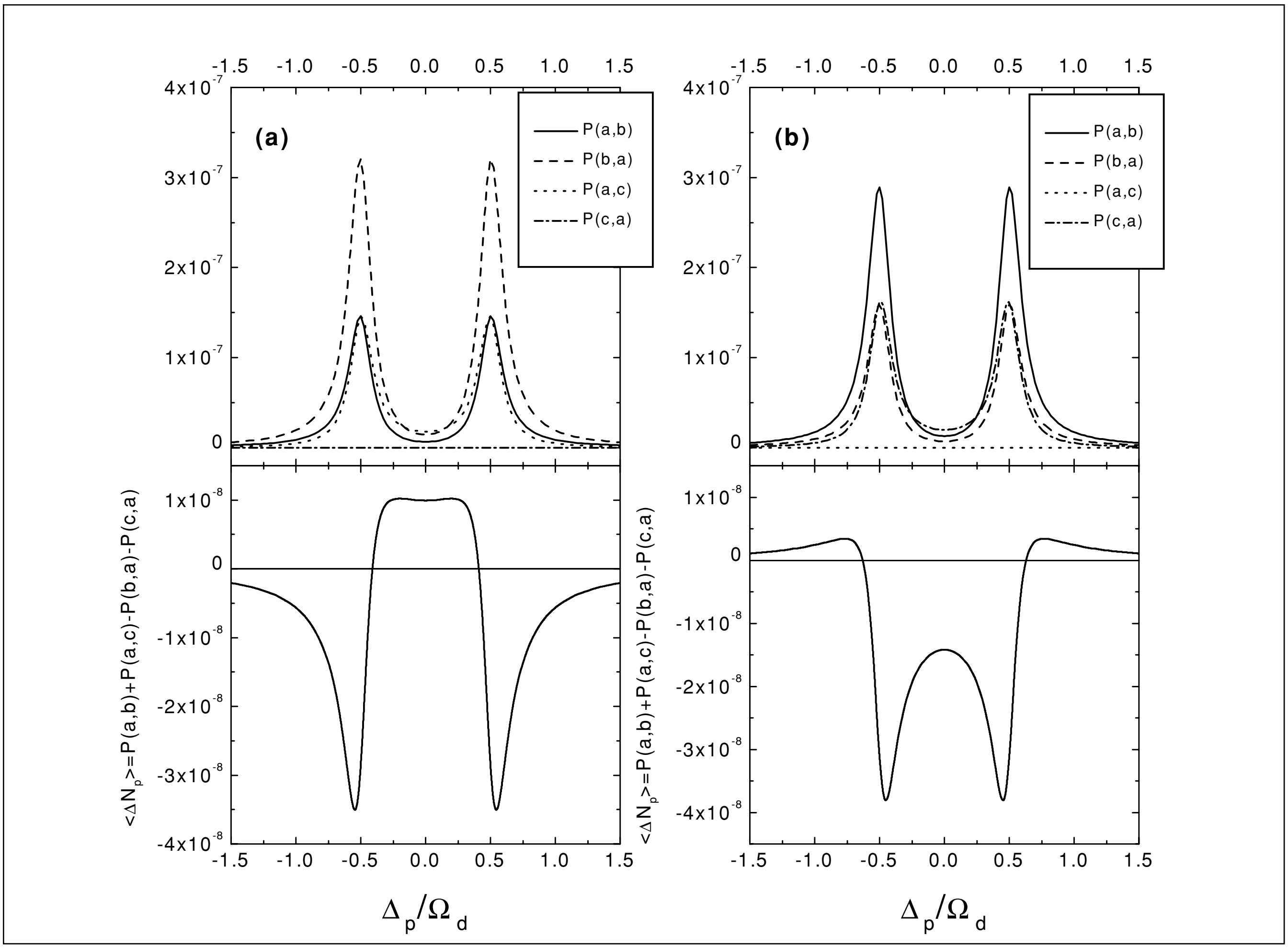

Figure 3. (a) *V*-scheme. (b) Cascade scheme. Upper plots show the probability that a random choice among all coherent evolution periods gives a one-photon gain process *P(a,b)*, a one-photon loss process *P(b,a)*, a two-photon gain process *P(a,c)*, and, finally, a two-photon loss process *P(c,a)*, as a function of the probe field detuning. The lower plots show the mean change of the probe field photon number per coherent evolution period. Curves in the lower parts of (a) and (b) are identical to the solid curves in figures 2(a) and (b), and in figures 1(c) and 1(f), respectively.

one-photon processes and two-photon processes. For instance, let us consider again the V and cascade schemes shown in Figures 1(a) and 1(d) with N_p and N_d the photon number of the probe and driving field, respectively. Moreover, let us call B_{ab} and B_{ba} the generalized Einstein B coefficients for one-photon gain ($\Delta N_p=+1$, $\Delta N_d=0$) and loss ($\Delta N_p=-1$, $\Delta N_d=0$) processes, and B_{ac} and B_{ca} the corresponding coefficients for two-photon gain ($\Delta N_p=+1$ with $\Delta N_d=-1$ for the V-scheme and $\Delta N_d=+1$ for the cascade scheme) and loss ($\Delta N_p=-1$ with $\Delta N_d=+1$ for the V-scheme and $\Delta N_d=-1$ for the cascade scheme) processes, respectively.

In both V and cascade schemes the amplification/attenuation of the probe field will be given by:

$$\frac{d}{dt}n_p = \hbar\omega_p n_p\left(\rho_{aa}B_{ab} + \rho_{aa}B_{ac} - \rho_{bb}B_{ba} - \rho_{cc}B_{ca}\right) \tag{6}$$

where n_p is the number of photons per unit volume in the probe mode. $n_p\rho_{aa}B_{ab}$ and $n_p\rho_{bb}B_{ba}$ account for the rates of one-photon gain and loss processes, while $n_p\rho_{aa}B_{ac}$ and $n_p\rho_{cc}B_{ca}$ account for the rates of two-photon gain and loss processes, respectively. It has been demonstrated by Mompart and Corbalán [30] that, assuming a strong and close to resonance driving field and in both cases that transition $|a\rangle - |c\rangle$ is electric dipole forbidden, the generalized Einstein B coefficients satisfy:

$$\frac{B_{ab}}{B_{ba}} \equiv 1 + \Delta B_{1p} = 1 + \frac{R_{bc} - R_{cb}}{R_{ba} + R_{cb}} \tag{7a}$$

$$\frac{B_{ac}}{B_{ca}} \equiv 1 + \Delta B_{2p} = 1 + \frac{R_{cb} - R_{bc}}{R_{bc}} \tag{7b}$$

where R_{ij} with $i,j=a,b,c$ corresponds to the rate of all possible incoherent processes taking out population from state $|i\rangle$ to state $|j\rangle$. Clearly, there is a symmetry breaking between the generalized Einstein B coefficients for one-photon processes and those for two-photon processes. The explicit value of this symmetry breaking is determined by the incoherent processes such that $\Delta B_{1p}\Delta B_{2p}\leq 0$. Notice also that the sign of $R_{bc}- R_{cb}$determines whether there is inversion or not in the driven transition. A detailed analysis of the properties of these generalized Einstein B coefficients can be found in Mompart and Corbalán [30].

Quantum Zeno effect in inversionless amplification

By using the quantum-jump formalism, AWI in three-level systems has only recently been discussed in terms of the quantum-Zeno effect, i.e., the inhibition of a coherent evolution due to a frequent measurement process. In the interaction of a three-level system with an intense laser field, the population evolution governed by the Rabi flopping plays the role of the nonlinear continuous evolution while irreversible processes, e.g., spontaneous emission, collisions and incoherent pumping, account for the wavefunction collapses or measurement processes. In the V-scheme, de Jong *et al*. [31] have shown that it is possible to use the quantum-jump formalism to analyze LWI in terms of the quantum Zeno effect. An extension to the rest of the three-level systems of the role of the quantum Zeno effect can be found in Mompart *et al*. [32].

Lasing without inversion with frequency up-conversion

Amplification and lasing without inversion has attracted much interest to generate high-frequency laser light where pumping methods, necessary for conventional lasing, are ineffective or difficult to implement. In all experimental reports on LWI, the coherence-inducing field, i.e., the driving field, had a frequency (ω_d) equal to or larger than that of the inversionless generated field (ω_p). Thus, the main experimental efforts on LWI are presently focused on the frequency up-conversion regime where $\omega_p>\omega_d$. Unfortunately, difficulties arise when the frequency up-conversion regime is addressed. Among these two should be highlighted [4,33]: (1) Doppler broadening due to atomic motion, and (2) propagation effects associated with driving field depletion along the active medium. With respect to the first problem, it has been shown theoretically for the V-type three-level system, that in the optical domain, the Doppler broadening of a vapor cell restricts LWI to frequency up-conversion ratios of $R=\omega_p / \omega_d \leq 2$ [34]. Nevertheless, the use of an atomic beam can considerably increase the frequency up-conversion ratio since, in this case, the transversal Doppler broadening is drastically reduced. In fact, two different atomic beam arrangements have recently been proposed in such a way that the probe-to-driving frequency ratio extends to values up to $R \approx 10$ [34]. Unfortunately, in the atomic beam configuration the probe gain per pass is very small due to the short interaction length [35].

In respect to propagation effects, Lukin *et al.* [33] have shown that a general characteristic of the frequency up-conversion regime is the rapid absorption of the driving field simultaneously to the small inversionless gain per unit length of the probe (or generated) laser field. In fact, Yelin *et al.* [36] have suggested the possibility of driving, with a microwave field, a weakly allowed transition while probing an adjacent transition with an optical field. Finally, a detailed analysis of the propagation effects in a Doppler-broadened V-type three-level system in rubidium can be founded in Mompart *et al*. [35].

Lasing without inversion in three-level systems without external coherent driving

In previous theoretical work, as well as in the three experimental demonstrations of LWI in atomic vapors [10-12], the coherence-generating driving field was imposed from the outside. Very recently, we have considered a Doppler-broadened three-level system placed in a doubly resonant cavity (Figure 4(a)) and driven only by external incoherent pump mechanisms [37]. As it is shown in Figures 4(b) and (c), these incoherent pump processes are characterized by unidirectional rates Λ and Λ'. The pump rate Λ' is considered large enough to invert the population at the driven transition and allows the generation of a laser field with Rabi frequency $\Omega_d(t)$ in a conventional way. This laser field induces the atomic coherence in the medium necessary to relax the population inversion condition in the adjacent transition and allows the generation of an inversionless laser field with Rabi frequency $\Omega_p(t)$. It must be noted that this system results advantageous especially in the frequency up-conversion regime, due to the large driving field intensity available. This is one of the ways to mitigate the negative role of the Doppler broadening.

In particular, we have applied our model for parameters appropriate to two real atomic systems: (1) a cascade scheme in atomic ^{138}Ba with frequency up-conversion ratio $R = 0.67$, and (2) a *V*-scheme in atomic ^{85}Rb with $R = 1.88$. These two cases are qualitatively different in that two-photon processes are responsible for inversionless gain in Case (1) while LWI in Case (2) arises from one-photon processes. In spite of the fact that the frequencies of the two laser fields are relatively different, our results show that dual-wavelength lasing extends up to Doppler broadening values typical for optical transitions of atoms in a vapor cell. Finally, let us mention that we have restricted our study to mainly the CW regime of our dual-wavelength laser. However, this system could exhibit interesting dynamic behavior due to the nonlinear coupling between the two laser fields.

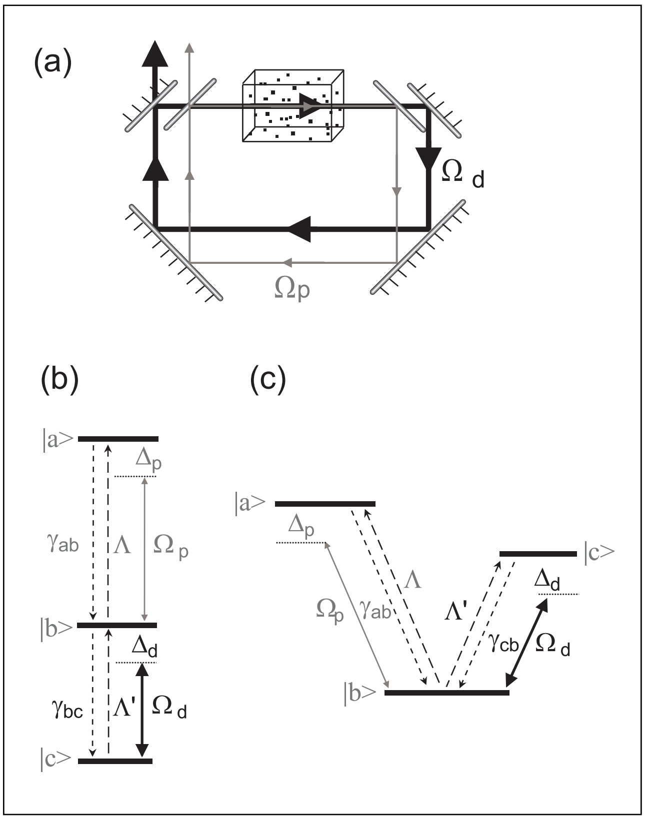

Figure 4. (a) Dual-wavelength laser with the atomic medium configuration being either (b) a cascade scheme or (c) a *V*-scheme.

LWI with standing wave driving

Most theoretical papers dealing with LWI have considered the case of a travelling wave driving field. Only very recently, the possibility of LWI in Doppler-broadened closed three-levels schemes with a standing wave driving has been addressed [38]. Thus, it has been shown that depending on the type of three-level system (closed folded or cascade schemes), multiphoton processes involving an even (odd) number of driving photons can contribute to probe amplification (absorption) or vice versa. These processes are only weakly affected by Doppler broadening for probe to driving frequency ratios R equal to integer numbers. Thus, we have recently demonstrated that closed folded (cascade) three-level schemes with even (odd) R numbers are suitable candidates for frequency up-conversion LWI in Doppler-broadened configurations.

In addition, the close phenomenon of electromagnetically induced transparency in Doppler-broadened three-level systems with resonant standing-wave driving has been also recently analyzed [39]. In this case, probe windows of transparency occur only for values of the probe-to-driving field frequency ratio R close to half-integer values. Thus, for optical transitions and typical values of Doppler broadening for atoms in a vapor cell, we have shown that for $R>1$ a standing wave field is appreciably more efficient than a travelling wave driving in inducing probe transparency.

Dynamical features of LWI

Continuous wave and self-pulsing LWI

A useful way to search for the possibility of LWI in three and multi-level atomic configurations consists of applying the techniques of nonlinear dynamics [40-43]. In order to find the LWI conditions, one performs a linear stability analysis of the system around the trivial non-lasing solution. For the CW emission regime, this approach is equivalent to the standard search for stationary solutions with the electric field amplitude different from zero. However, the dynamical point of view allows one to find conditions when LWI takes place in a self-pulsing regime. From the analysis of resonant closed three-level systems, it has been shown that near lasing threshold V and Λ systems yield CW LWI, while cascade systems can give rise to self-pulsing LWI. These results are easily interpreted in view of the gain spectra shown in Figures 2. Thus, for the *V*-system with the laser cavity tuned to resonance, i.e., $\Delta_c=\Delta_p=0$ with Δ_c the cavity detuning, the cavity mode experiences gain (Figure 2(a)). When this gain overcomes cavity losses one has CW lasing. Conversely, the cascade system does not exhibit gain at $\Delta_c=\Delta_p=0$ (Fig. 2(b)) but at two sidebands symmetrically located about line centre, at approximately $\Delta_p=\pm 0.7\Omega_d \equiv \pm(\Delta_p)_t$. Thus, the resonant cavity mode can be amplified in a self-pulsing regime, with its intensity modulated at the angular beating frequency $2(\Delta_p)_t$. An analysis analogous to the previous one shows that LWI based on a double-Λ system can generate both CW and self-pulsing emission depending on the parameter values [44,45]. The underlying mechanism also involves coherent population trapping [1], and then the pulsed emission has an origin similar to that in a conventional laser with large gain and cavity losses.

Giant pulse lasing

As discussed previously, the presence of an external coherent field acting on one transition of a three-level medium modifies substantially the conditions for laser oscillation in the other transition, which can occur even in the absence of population inversion. This reverse phenomenon is known as population inversion without lasing (IWL). Recently, we proposed the application of the IWL phenomenon for generating giant pulses of laser light [46]. The proposed method is an alternative to the standard Q-switching technique for generating pulses of short duration, e.g., 10^{-7}-10^{-8} sec, and relatively high peak power, e.g., 10^6-10^7 W, the giant pulses.

Consider the cascade scheme of Figure 1(d) within a ring laser cavity prepared to generate a laser field, the giant pulse, in the upper transition $|a\rangle - |b\rangle$. An incoherent continuous pump mechanism $\Lambda_{b\to a}$ pumps population from level $|b\rangle$ into level $|a\rangle$ and, therefore, allows it to invert the population of the upper transition. In addition, an external pulsed laser $\Omega_d(t)$ drives the lower transition $|b\rangle - |c\rangle$. In what follows, for simplicity, we will consider the completely resonant case, i.e., Δ_c=Δ_p=Δ_d=0. For appropriate parameter values (Figure 5), a linear stability analysis shows that in the presence of the driving field the nonlasing solution is stable, although the steady-state population inversion is well above the threshold inversion for lasing without the driving field. Thus, as shown in Figure 5, during the first 225 μs the presence of the driving field (Figure 5(a)) with an intensity of 3.7 W·cm^{-2}, prevents laser oscillation and a large population inversion accumulates (Figure 5(b)). The dotted line in this figure represents the threshold population inversion in the absence of the external laser field, i.e., $n_{th}(\Omega_d$=0). After the population inversion saturates, the driving field is switched off for 25 μs and a giant pulse develops (Figure 5(c)) since then the population inversion is well above the threshold population inversion (dotted

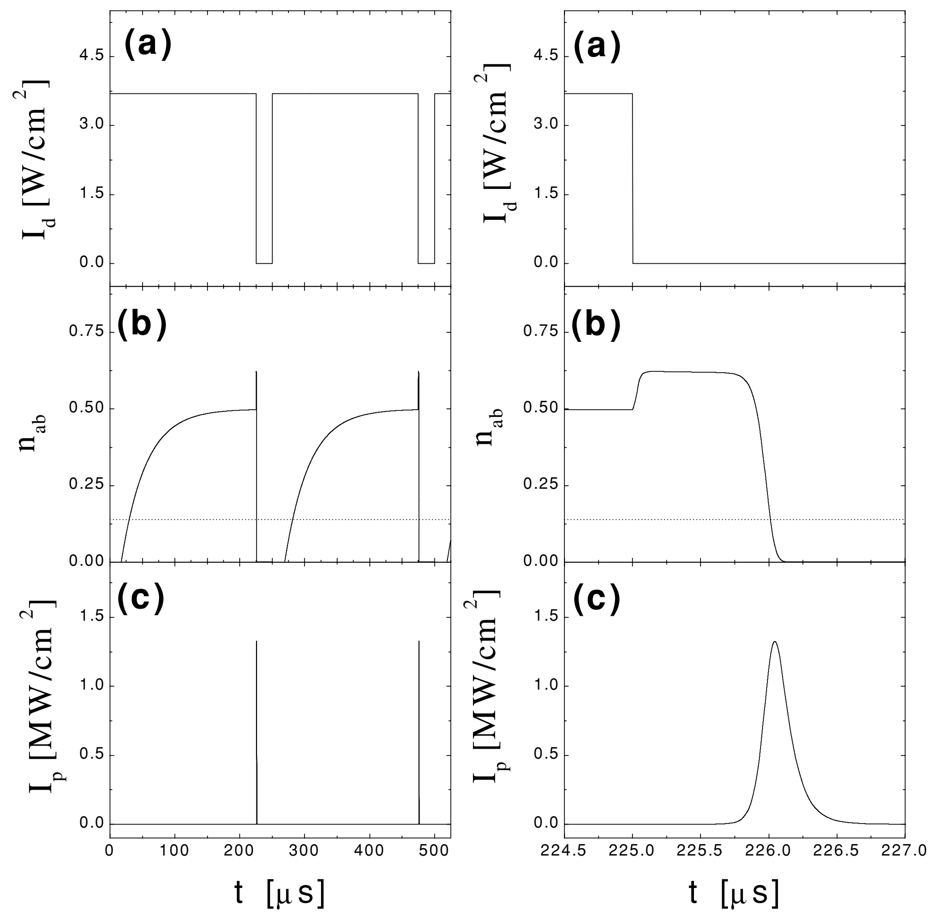

Figure 5. Time evolution of (a) driving field intensity, (b) population difference in the laser transition (n_{ab}≡ρ_{aa}–ρ_{bb}), and (c) intensity of the generated laser pulse. The parameters are: γ_{ab}=10kHz, γ_{bc}=50kHz, $\Lambda_{b\to a}$=50kHz, Ω_d=50MHz, κ=5MHz, g=900MHz^2, μ_{ab}=10^{-31} C·cm, and μ_{bc}=10^{-28} C·cm) with κ the cavity losses, g the unsaturated gain parameter of the lasing transition, and μ_{ab} (μ_{bc}) the electric dipole moment of the lasing (driven) transition. The dotted line in (b) represents the threshold population inversion for laser oscillation in the absence of the driving field, i.e., $n_{th}(\Omega_d$=0). Pictures in the right column are and enlarged view of the corresponding time evolution shown at the left column.

line). For the parameters used, the peak intensity of the laser pulse gives $(I_p)^{peak}$=1.33 MW·cm^{-2} MW·cm^{-2} with a pulse width at half maximum about 210 ns and an integrated pulse energy of 0.33 mJ·cm^{-2}. Note that the pulse duration is similar to that of the longest pulses from traditional Q-switched systems. It should be remarked that a laser field with a power of a few watts is able to control the generation of laser pulses at a few megawatts of peak power.

Summary

We have reviewed the interest, the state-of-the art and the basic physics of lasing without population inversion (LWI). Different theoretical approaches to LWI have been discussed. We have also analyzed the origin of LWI in coherently driven three-level systems by using density matrix and quantum-jump formalisms. We summarized the main difficulties towards x-ray lasing and described different proposals to overcome these difficulties. Finally, some dynamical features of these lasers, such as, self-pulsing lasing without inversion and the generation of giant pulses of laser light have been investigated.

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During the last two decades, researchers led by Prof. Corbalán at the Optics Group of the Autonomous University of Barcelona have focused on theoretical and experimental aspects of coherently pumped gas lasers, laser spectroscopy and nonlinear laser dynamics. Since 1995, this research has focused mainly on lasing without population inversion and other atomic coherence effects. Ramón Corbalán is a professor of Quantum Optics at the Autonomous University of Barcelona.

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