The errors and animadversions of Honest Isaac Newton*

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Most of you know that Newton created a unified theory of celestial and terrestrial mechanics; that he explained the ocean tides, that he showed sunlight to be a mixture of all the colors of the rainbow; that he invented (or co-invented) calculus; that he designed and built the first reflecting telescopes; that he was showered with honors during his lifetime, even serving his government as Master of the Mint; and that he is buried beneath one of the most obscenely flamboyant tombs in Westminster Abbey. All this and more is true, which explains Alexander Pope's often quoted couplet:

"Nature and Nature's laws long lie hid in Night God said 'Let Newton be,' and all was light."

Pope honored the man many regard as the greatest scientist who ever lived, of whom, more recently and with mischievous admiration, Robert Frost asked: "How many apples fell on Newton's head before he took the hint?" Yet, I suspect that few of you are familiar with the darker side to Isaac Newton, the less savory aspects of his life that led Aldous Huxley to express the view that "as a man [Newton] was a failure, as a monster he was superb."

Every scientist makes mistakes, takes missteps and commits misdeeds. Galileo could not abide Kepler's elliptical planetary orbits, while Kepler himself cast horoscopes for a living. John Dalton, the British father of chemistry, missed the 2 in H2O, just as Antoine Lavoisier, 'le père français de la chimie', erred by listing heat and light as chemical elements. Ernest Rutherford may have discovered the tiny nuclei within atoms, but

he regarded the quest for nuclear energy as mere moonshine. The great Niels Bohr was twice prepared to abandon the law of energy conservation and Einstein refused to see Nature as a quantum game of chance. More recently, Richard Feynman couldn't bring himself to call a quark a quark, while T.D. Lee and C.N. Yang, who once shared a Nobel Prize, barely speak to one another today. Isaac Newton was no exception. Not by a long shot! Here I shall offer a sampling of Newton's many transgressions, social, scientific and religious.

Newton the tyrant

"If I have seen further," Newton famously wrote to his contemporary, Robert Hooke, "it is by standing upon the shoulders of giants." No expression of humility was this! These men were scientific rivals and bitter enemies. Hooke's frequent demands for acknowledgement of his own significant contributions to optics and mechanics infuriated Newton, who insisted that Hooke's claims were without merit. Physicist and author Jeremy Bernstein wrote: "On the face of it, Newton appears to be calling Hooke a giant, but the hyperbole is, after all, addressed to a [quote] 'crooked little man'." [The

Encyclopedia Britannica describes Hooke's appearance as "but a sorry show. His figure was crooked [and] his limbs shrunken", but the amatory exploits Hooke describes in his diaries suggest that he was quite attractive to women as a young man.] Bernstein concludes: "It was not so much that Newton wanted to stand on Hooke's shoulders, but rather to step on his head."

Most physics students know little of Hooke aside from his law about springs, although, as the physicist-historian of science Michael Nauenberg notes, "Hooke was one of the most prolific and inventive scientists of all time, [making] fundamental contributions to virtually all branches of science." It is true that

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Hooke lacked Newton's mathematical sophistication. When he asked Newton for mathematical help, "Newton solved his problem, but never acknowledged Hooke's seminal contributions." Instead, Newton wrote that "[Hooke] has done nothing, and yet written... as if he knew all, [leaving] what remained to be determined to the drudgery of calculations, excusing himself from that labor by reason of his other business, whereas he should rather have excused himself by reason of his inability." In fact, Hooke had explained to Newton how Kepler's area law (conservation of angular momentum) follows for any central force. Without this proof, Newton's *Principia* could not have been written. "The theory of planetary motions," Nauenberg concludes, "should be recognized as a remarkable joint scientific achievement of Newton and Hooke."

Was it a coincidence that Newton was elected President of the Royal Society in the very year that Hooke died? Is it true that Newton destroyed the only known portrait of Robert Hooke? The mere existence of such a vile rumor informs us of Newton's undying animus toward his rival.

Hooke needed mathematical advice from the master for good reason. Newton's analysis depended on a mathematical tool he had developed for that very purpose: the theory of fluxions, or what we now know as calculus. Once again, Newton's demand for every shred of credit led to a nasty priority fight, one between Newton and the German mathematician and philosopher Gottfried Wilhelm von Leibniz. Newton may have taken the first giant steps toward calculus, but much of what we know about his later contributions is indirect. Newton was obsessively secretive, reluctant to publish, averse to public speaking, and sometimes hid his ideas as coded Latin anagrams. (Some contemporary psychohistorians argue that he suffered from an affect disorder known as Asberger's syndrome [or Asperger's syndrome?].) His greatest accomplishment, the mathematical analysis of planetary motion in the Principia, uses ingenious and elaborate geometrical arguments rather than simpler ones based on calculus. Newton once said to a friend that he purposely made it difficult "to avoid being bated by little smatterers in mathematics."

Today it is generally recognized that both men made essential contributions to mathematics. Leibniz' genius was to have developed a concise, transparent, and readily transportable notation for the principal operations of calculus: the familiar dy/ dx for differentiation and R sign for integration. Newton complained that "Leibniz named the things that I invented." Late in the 1690s, Newton's followers accused Leibniz of plagiarizing Newton's ideas. In 1711, Leibniz appealed to the Royal Society-of which he was a member, and Newton its then-president-for redress from these serious and almost certainly unjustified charges. None was forthcoming. Morris Kline writes: "Continental mathematicians sided with Leibniz, while English mathematicians defended Newton. The two groups became unfriendly and even bitter toward each other... Because Newton... used geometrical methods, the English continued to use mainly geometry for about a hundred years after his death. The Continentals took up Leibniz' analytical methods... [which] proved to be far more effective; so not only did the English mathematicians fall behind, but mathematics was deprived of contributions... of some of the ablest minds." Once again, Newton could have been more gracious, and he could have had a more salutory effect on British mathematics.

We have seen how Newton's relations with Hooke and Leibniz were less than cordial. These were not isolated instances of Newton's strained interactions with his peers. William Whiston, like Newton, followed the then-heretical faith of Arianism, today's Unitarianism, which rejects the doctrine of the trinity and denies the divinity of Jesus. Whiston was Newton's assistant, later becoming his chosen successor as the Lucasian Professor at Cambridge (a post now occupied by Stephen Hawking). About a decade afterward, Whiston was expelled from Cambridge for professing the heretical beliefs that Newton never abandoned, but had learned to conceal. Later on, when Whiston was put forward for membership in the Royal Society, Newton (as its President) squelched the appointment. Long afterward. Whiston would explain: "If the reader desires to know the reason of Sir Isaac Newton's unwillingness to have me a member, he must take notice that [Newton] made me his successor; so did I enjoy a large portion of his favor for twenty years together. But he then perceiving that I could not do as his other darling friends did, that is, learn of him, without contradicting him, he could not, in his old age, bear such contradiction; and so he was afraid of me the last thirteen years of his life.... He was of the most fearful, cautious, and suspicious temper, that I ever knew."

The title of this section alludes to the book "Newton's Tyranny: The Suppressed Scientific Discoveries of Stephen Gray and John Flamsteed" by David and Stephen Clark. Flamsteed, an accomplished cartographer and England's first Astronomer Royal, was charged to produce a catalog of the stars. He collected vast amounts of data for many years, largely at his own expense because Newton failed to secure the promised funding. Well before the project was completed, and despite Flamsteed's strong objections, Newton had the unfinished manuscript published so that he could make use of the data. Flamsteed managed to acquire and burn hundreds of copies of the unauthorized document and even brought Newton to court. In retaliation, Newton had Flamsteed expelled from the Royal Society.

Introductory physics textbooks describe how Stephen Gray, an impecunious dyer and an amateur scientist, discovered how electric charge could be conveyed from one body to another distant body through what were later called electrical conductors. Gray had the misfortune to be Flamsteed's correspondent and confidante. The friend of my enemy is my enemy as well, Newton must have felt, because he delayed and opposed the publication of Grav's seminal research. Only after Newton's death was Gray awarded the recognition he deserved. "Those that have begun to do ill things never blush to do worse to secure themselves," Flamsteed later wrote, "Sly Newton had still more to do and was ready at coining new excuses and pretences to cover his disingenuous and malicious practices... Honest Sir Isaac Newton (to use his own words) would have all things in his own power, to spoil or sink them..." Flamsteed, it seems, was almost as intemperate and uncompromising as Newton himself.

Newton's scientific arrogance

Let us examine some of Newton's serious scientific blunders, exaggerations and obfuscations. Newton learned of Robert Boyle's great discovery that "there is a spring to the air." (In fact, it was our friend, Robert Hooke, as Boyle's assistant, who carried out the experiments.) Imagine a cylinder, the air within confined by a freely moveable piston. Push the piston inward and the compressed air within resists; pull it outward and it also resists. Thus, the device behaves like a rubber band or a spring. These observations are encapsulated as a quantitative eponymous law: that the pressure of a quantity of gas is proportional to its density (if—as we now know—the gas is kept a constant temperature).

Newton sought to deduce Boyle's Law from first principles. His argument went as follows: He imagined a gas to consist of a vast multitude of tiny particles. So far, so good. But his gas particles were at rest, "[repelling] each other with forces that are reciprocally proportional to the distances of their centers." He showed that such a system of particles would "compose an elastic fluid whose density is as the compression." That is, Boyle's Law followed from the mutual repulsion of air molecules. Newton did hedge a bit: "Whether elastic fluids do really consist of particles so repelling each other is a physical question. We have here demonstrated mathematically the property of fluids consisting of particles of this kind, that hence philosophers may take occasion to discuss that question."

Philosophers (as scientists were then known) have done just that. Newton's tiny particles do exist: they are now called molecules. His conjectured repulsive force between them, however, would lead to all sorts of problems. Fortunately, there is no such force. Newton's explanation of Boyle's law was dead wrong.

A decade after Newton's death, the Swiss scientist Daniel Bernouilli pinpointed the correct principles underlying Boyle's Law. Bernouilli's hypothetical corpuscles were not stationary, as Newton thought, nor were they subject to any mysterious forces. They moved freely and erratically with random velocities, occasionally suffering elastic impacts with one another or with the containing vessel. But this correct and brilliant insight would languish for well over a century. "Why was [Bernouilli's] theory forgotten?" asked the contemporary historian of science Stephen Brush. It was because "Newton's theory... had been firmly established as the explanation of Boyle's Law, both by the reputation of Newton and by its simplicity... Once the Newtonians were in power, no prudent scientist (at least in England) would dare to contradict the real or supposed opinions of the 'autocrat of science' until late in the 19th century."

Newton's flawed understanding of the elasticity of air did not prevent him from seeking to understand the propagation of sound, which he wrote "can be nothing else but pulses of the air propagated through it," that is, sound waves. From an ingenious but rather difficult to understand argument, he deduced a simple formula for the speed of sound in air:

$$V = \sqrt{\frac{\text{Pressure}}{\text{Density}}}$$

But for a multiplicative factor that is not so different from unity, Newton's result is correct. It stands as a triumph of Newton's physical intuition. He could not have known that sound propagation is not isothermal, and that the correct result involves an additional thermodynamic parameter.

Newton would not be satisfied with a result that was only approximately correct. He was compelled to swindle his way to exactitude. Of the preceding calculation, he wrote, "We have made no allowance for the crassitude of the solid particles of the air, by which the sound is propagated instantaneously." (In Newton's time 'crassitude' meant 'thickness'. Newton was arguing that the physical size of air molecules somehow enhanced the velocity of sound.) With this dreadfully spurious argument, Newton augmented his estimate of the speed of sound by about 10%, but his modified result was still significantly smaller than its known speed. He then appealed to an even more absurd argument: "The vapors floating in the air being of another spring, and a different tone, will hardly, if at all, partake of the motion of the true air in which the sounds are propagated.... So if the atmosphere consists of 10 parts of true air and one part of vapors [If indeed!], the motion of sounds will be swifter by the subduplicate ratio [that is, the square root] of 11 to 10..." Having resorted to the alleged crassitude of air and its presumed contamination by inert foreign vapors, Newton triumphantly boasted of an answer that agreed precisely with the experimentally determined speed of sound.

Of these manifestly fraudulent arguments, Newton scholar Richard Westfall wrote: "Not the least part of the *Principia*'s persuasiveness was its deliberate pretense to a degree of precision quite beyond its legitimate claim. If the *Principia* established the quantitative pattern of modern science, it equally suggested a less sublime truth: that no one can manipulate the fudge factor quite so effectively as the master mathematician himself."

Another instance of Newton's flawed brilliance concerns his investigation of ocean tides, which he showed to result from the gravitational attractions of the moon and the sun. "The force of the moon to move the sea," he correctly notes in the *Principia*, "varies inversely as the cube of its distance from the earth." Newton never described how he got this result, and his contemporaries were at a loss to understand it. Using some rough tidal data, Newton found (quite wrongly) that "the moon is more dense than the earth itself." In 1713, he found the ratio of the masses of the earth and moon to be 39.371, whereas its actual value is about 81. Here, Newton asserts five decimal place precision for a result that is wrong by over a factor of two!

Even more astonishing was Newton's so-called 'Moon Test': his claim that the moon, if its orbital motion were stopped by some magical agency, would fall toward earth a distance of 14.7706353 feet in one minute: nine significant figures, of which only the first two are correct! As Westfall writes, "Newton doctored still another computation in his effort to create an illusion of great accuracy." And yet, let us not forget that Isaac Newton was the first person to provide a correct explanation for why there are two high tides each day. Many of Newton's

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greatest accomplishments, such as his calculation of the speed of sound and his studies of the tides, are beclouded by his tendency toward specious argumentation and arrogant exaggeration.

Newton made enormous contributions to the science of light. He developed a mathematically precise theory of geometrical optics, which led, among other things, to his invention of the relecting telescope. He showed how sunlight, passing through a glass prism, is dissected into its constituent and immutable colors, and how these could be recombined into white light. He explained these phenomena in terms of the varying 'refrangibility' of the different colors of light as they pass through a transparent material. (Today we would say that the index of refraction of a material medium is frequency dependent.) Newton related the apparent color of a body to the degree that it reflects or absorbs the different colors of ambient light. His careful measurements of the curious colored patterns produced by light reflected from a lens atop a glass plate—which are known as Newton's Rings, although they were first described by Hooke-suggested to him a flawed analogy between light and sound, a kind of harmony of color. Had Newton been more open-minded, these studies could and should have led him away from his corpuscular theory toward a more satisfactory wave theory of light, such as had been advocated by Hooke and Huygens.

But the notion of light as a stream of material particles was absolutely central to Newton's scientific philosophy: "Are not the Rays of Light very small Bodies emitted by shining Substances? For such Bodies will pass through uniform Mediums in right Lines without bending into the Shadow, which is the nature of Rays of Light. They will also be capable of several Properties and will be able to preserve their Properties unchanged, which is another Condition of the Rays of Light." Newton was stubbornly unwilling to acknowledge the growing empirical evidence, much of it his own, that light, under all circumstances then known, behaves as if it were a wave phenomenon. This was the first and foremost of his several optical errors.

Snell's law, familiar to all students of elementary physics, describes the bending (or refraction) of light rays as they enter or leave a transparent medium. Newton attributed the phenomenon to hypothetical attractive forces acting between particles of light and those of matter. Hypothetical forces yet again! Flushed with pride for his universal law of gravitational attraction, Newton imagined other forces to lurk everywhere, just as my government imagined Iraqi weapons of mass destruction: Newton proposed a repulsive force to explain Boyle's law, and an attractive force to explain Snell's law. Newton's widely accepted theory of refraction required light to travel faster through glass than air. Not until the mid 1800s was this prediction falsified by experiment, putting a full stop to Newton's corpuscular theory. It soon became clear to everyone that light is an electromagnetic wave, or so it seemed until the 20th century, when we would learn that light is neither wave nor particle. The seemingly paradoxical nature of quantum mechanics, however, is the subject of quite another lecture.

Newton's belief in an exact correspondence between the color of light and its refrangibility led him to the false conclusion that a measurement of the index of refraction of one color would determine the index for all colors. He wrote: "Although I have not yet derived the certainty of this proposition from experiments, nevertheless I do not doubt that it will satisfy all of them which it is possible to do... meanwhile I am content to assume it gratuitously." Newton scholar Alan E. Shapiro claims that: "When [Newton] finally chose the linear dispersion law in the Opticks, [one of two mutually inconsistent laws he had at times proposed] he supported it with fabricated experimental evidence." Incidentally, Newton's theory led him to conclude, indeed, to insist, that an achromatic lens (one corrected for chromatic aberration) could not possibly be fabricated. Not so! It turned out that just such a lens was designed and created soon after Newton's death-by the British gentleman barrister Chester More Hall... as a hobby in his spare time!

Reason versus revelation—Newton as creationist

Isaac Newton was not only a physicist and a mathematician. He devoted at least as much of his time to alchemical experimentation, religious scholarship, and the study of mythology as history, especially biblical chronology. Wisely, Newton never published most of this nonsense. In his book "Isaac Newton, Historian" Frank Manuel writes: "Although the content of [Newton's unpublished papers] have been generally known for some time... they were usually avoided as heretical, nonsensical or 'mystical,' the dark side of the hero..." The renowned French scientists Laplace and Biot "took a dim view of these writings as dangerous to the course of science..." Perhaps, as Marjorie Nicholson says, "Newton would have preferred the proud title of 'theologian' to that of 'philosopher' or 'scientist'." Or perhaps, as economist John Maynard Keynes suggests, he would better be described as a magician, "because he looked on the whole universe and all that is in it as a riddle, as a secret which could be read by applying pure thought to certain evidence, certain mythic clues which God had left about the world... He believed that these clues were to be found... partly in certain papers and traditions handed down by the brethren in an unbroken chain back to the original cryptic revelation in Babylonia." Today, we might call a person with such beliefs a creation scientist.

At age thirty, Newton turned his analytical mind to Christian theology. With the passion of a rebel, Westfall writes, "Newton convinced himself that the received tradition was a fraud perpetrated by evil men in the fourth century who, for their own selfish purposes, had willfully corrupted the entire heritage. Newton's determination to unmask this ancient crime... absorbed virtually all of his time for the following fifteen years, until a visit from Edmund Halley [of Halley's comet fame] started the investigation that resulted in the 'Principia' and altered the tenor of his existence." We might not fault

Newton for his closet heresies, nor for his fanciful deduction that the Argonauts sallied forth in search of the Golden Fleece

in 939 BC, nor for his prediction that the Final Judgement would take place in 1867. But we shall mention some of his monumentally hubristic attempts to reconcile his scientific researches with his firm belief in an active and attentive Deity, one whose study, "from the appearances of things, does certainly belong to Natural Philosophy."

Newton found evidence for the existence of God throughout the cosmos. How else could the primal matter of the universe have organized itself into luminous stars and opaque bodies like the planets and their satellites? How else could the orbits of the planets be made properly spaced and nearly circular so as to ensure the stability of the solar system? "This most beautiful system of the sun, planets and comets could only proceed through the counsel and dominion of an intelligent and powerful Being," he wrote. Indeed, Newton's God was very much a scientist in Newton's own image: "To make this system with all its motions required a Cause which understood [the various masses, distances, and velocities] of the sun and planets.... And to compare and adjust all these things... argues that Cause to be not blind and fortuitous, but very well skilled in Mechanics and Geometry." Voltaire said it more elegantly: "L'horloge implique l'horloger", a clock implies a clockmaker. But here lurked a grave theological threat.

Although God may have set the universe in motion at the time of Creation, could it be that He was no longer needed? Would not Newton's own laws suffice to describe an eternal clockwork universe? "To maintain a role for Providence," wrote Newton scholar David Kubrin, "meant providing essential chores for God to perform, so that He did not rule over a universe able to exist without Him."

Newton knew that the solar system would endure for 'many ages' in its present form, but he also believed that the small irregularities in planetary motions and the gravitational effects of the planets on one another, "will be apt to increase, til this System wants a Reformation," and again, "Motion is more apt to be lost than got, and it is always upon the Decay... Seeing therefore that the amount of Motion which we find in the World is always decreasing, there is a necessity of conserving and recruiting it by active principles." Eventually the clock would run down and the world would tend to dissolution, unless the Creator would intervene. Having established the necessity for God's continuing attention to the proper running of His universe, Newton sought the rational means by which the Godly reformations were to be accomplished.

At one point, Newton appeals to an all-pervading aetherial fluid that is perpetually circulated so as to restore Earth's rivers and atmosphere, "and as the Earth, so perhaps may the Sun imbibe this Spirit copiously to preserve his Shining, and keep the Planets from receding further from him... the vast aethereal Spaces between us and the stars [being] a sufficient repository for this food for the Sun and Planets."

Elsewhere, Newton decided upon on comets as the Lord's agents of Earth's regeneration. The appearance of cometary tails suggested that their substance is being "scattered through the whole heavens, and by little and little... attracted toward the planets... and mixed with their atmosphere... [so as to replenish] the fluids spent upon vegetation and putrefaction... The

fluids, if they are not supplied from without, must be in continual decrease, and quite fail at last. I suspect that it is chiefly from comets that spirit comes [which is] so much required to sustain the life of all things with us."

Likewise, Newton enlisted comets to restore and renew the sun and the stars: The comet of 1680 (Halley's comet, an earlier passage of which Halley claimed to have been responsible for the Biblical Flood,) "must have suffered some resistance and retardation; and therefore... will at last fall down upon the body of the sun... So fixed stars, that have been gradually wasted by the light and vapors emitted from them for a long time, may be [renewed] by comets that fall upon them; and from this fresh supply of new fuel those old stars, acquiring new splendor, may pass for new stars." Indeed, Newton conjectured that the supernovae observed by Tycho and Kepler had resulted from such cometary impacts. As Kubrin writes, Newton "was able to account for both the periodic recruiting of motion and activity for the sun and planets and the 'reformations' necessary to reset the system from time to time. [Although] such revolutions were accomplished by mechanical means, Newton believed that they were under divine supervision."

Kubrin finds that "Newton's ideas implied the existence of Earths before this one, with the presence of races of man before Adam, and it seemed probable [to Newton] that the creation of Earth described in Genesis was only one in a series of Creations." (The Aztecs, incidentally, held a similar but more explicit view: that the world had been created and destroyed four times before its current incarnation.) As for the future, Newton helpfully suggested that "the Satellites of Jupiter can take the places of Earth, Venus, Mars if they are destroyed, and be held in reserve for a new Creation." Dangerous thoughts these! Newton never dared publish his unorthodox cosmogony, which act would certainly have jeopardized his position, and perhaps his life.

In summary, Newton certainly was a creationist in the sense of seeking, as best he could, scientific support for the account of Creation as given in the bible. Had he lived today, would he be a creationist in its present sense? Would Newton have continued to believe the biblical story of Creation, given the evidence we now have for the gradual evolution of the features and creatures of our planet? Or, for the birth of the entire visible universe 14 billion years ago in the Hot Big Bang? I doubt it, but we shall never know. What we do know is that Newton, warts and all, was surely one of the greatest intellects the world has known.

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