

English summaries

Marc Noy

The great theorem of modern combinatorics

If you ask an expert which is the star result in combinatorics in the last decades, it is likely that the answer is Robertson and Seymour's Minor Theorem. This result, one of the greatest achievements from last century mathematics, claims the following: if \mathcal{G} is a class of graphs closed under taking minors, then there is a *finite* number of obstructions determining if a graph is in \mathcal{G} . The classical example is Kuratowski's theorem: if \mathcal{G} is the class of planar graphs, the obstructions are the graphs K_5 and $K_{3,3}$. The proof spans about twenty papers with a total of more than 500 pages. In this article we explain the content of the theorem and give an idea of its implications, its relations with algorithmics and logic, the fundamental tools it has given rise to, and its proof.

Keywords: planar graph, graphs in surfaces, graph minors, tree-width, computational complexity.

MSC2010 Subject Classification: 05C83.

Eulàlia Nualart

Integration by parts formula in a Gaussian space and its applications

In the 70's, the french mathematician Paul Malliavin revolutionized the probability theory when he introduced the stochastic calculus of variations that today has his name. Malliavin constructed a differentiable structure in a Gaussian space such that the Itô integral was a differentiable object. His main motivation

was to use this theory in order to provide a probabilistic proof of Hörmander's theorem for second order hypoelliptic operators. One of the key tools of this differential stochastic calculus is its integration by parts formula that concerns two operators, the derivative and its adjoint, named the Skorohod integral. We will introduce the basic notions of the Malliavin calculus, and we will give some of its applications to three different -but very related- areas of mathematics, which are probability, statistics and mathematical finance.

Keywords: Integration by parts formula, Malliavin derivative, Skorohod integral, Malliavin calculus and its applications.

MSC2010 Subject Classification: Primary: 60H07, 60J60; Secondary: 62F12, 91B28.

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Fermat's Method of Quadrature

The *Treatise on Quadrature* of Fermat (c. 1659), besides containing the first known proof of the computation of the area under a higher parabola, $\int x^{+m/n} dx$, or under a higher hyperbola, $\int x^{-m/n} dx$ — with the appropriate limits of integration in each case — has a second part which was mostly unnoticed by Fermat's contemporaries. This second part of the *Treatise* is obscure and difficult to read. In it Fermat reduced the quadrature of a great number of algebraic curves to the quadrature of known curves: the higher parabolas and hyperbolas of the first part of the paper. Others, he reduced to the quadrature of the circle. We shall see how the clever use of two procedures, quite novel at the time: the change of variables and a particular case of the formula of integration by parts, provide Fermat with the necessary tools to square — quite easily — as well-known curves as the folium of Descartes, the cissoid of Diocles or the witch of Agnesi.

Keywords: history of mathematics, quadratures, integration methods.

MSC2010 Subject Classification: 01A45, 26-03, 26B15, 51M25.

Oriol Serra

The Brunn-Minkowski inequality

This paper gives a general overview of the Brunn-Minkowski inequality, which relates the measure of the sum of two sets with the measure of the summands, and some of its variants, particularly its discrete versions. This classical inequality, strongly related to the isoperimetric inequality, has numerous applications in several areas of mathematics and computer science.

Keywords: Brunn-Minkowski inequality, isoperimetric inequalities.

MSC2010 Subject Classification: 05D, 11B, 11H, 52A, 68Q25, 90C27.
