

English summaries

Elisenda Feliu

Polynomials, polytopes, and steady states of reaction networks

In this paper we introduce the theory of the study of steady states of reaction networks, and we focus on examples from molecular biology. Steady states are the positive solutions to a system of polynomial equations containing numerous parameters. One of the objectives of the theory is to study steady states as a function of parameters, and, in particular, to determine their number. These problems can be solved using tools from real algebraic geometry and computational algebra, but the specific characteristics of the systems stemming from reaction networks have allowed more in-depth findings to be obtained. In this article we explain some of the recent effective results in this area, where the study of the system and of the parameter regions is possible thanks to the examination of the geometry of an associated polytope.

Keywords: reaction network, multistationarity, bistability, Newton polytope, positivity, real algebraic geometry.

MSC2020 Subject Classification: 92C45, 34E15, 80A30, 13P10.

David Rojas

Oscillators at resonance

An oscillator is isochronous if all motions are periodic with a common period. When the system is forced by a time-dependent periodic perturbation with the same period, the dynamics may change drastically and the phenomenon

of resonance can appear. In this article we will study which properties the perturbations must fulfil in order to obtain unbounded solutions. We will consider different oscillators, from harmonic to nonlinear generalizations, and we will set out a number of remarks about the concept of auto-parametric resonance.

Keywords: oscillator, resonance, perturbation, isochrony.

MSC2020 Subject Classification: 34C10, 34C15, 34D05, 34D10, 34D23.

Tomás Sanz-Perela

Why do we like music? A mathematical answer

Why do we like music? Why do we feel that the sound produced by one or more piano keys is music and yet we call the sound that a glass makes when falling to the ground noise? Why do we hear the same note played by a flute or a clarinet differently? And why, without having studied music, are we able to distinguish a person who has just started studying the violin and plays out of tune from an experienced one?

In this article, we give answers to these questions using mathematics as the main tool. To do so, our starting point will be the wave equation, which will allow us to understand the main properties of the sound produced by musical instruments. Based on this knowledge we will be able to understand the ideas which, throughout history, have been behind the construction of musical scales, which form the basis of most of the music we are acquainted with nowadays. Finally, we will study the concepts of *dissonance* and *consonance* from a mathematical perspective, and we will gain a better understanding about why some sounds are more pleasant than others.

Keywords: wave equation, harmonic spectrum, Fourier series, musical scales, dissonance.

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