

English summaries

Pere Ara

The Banach-Tarski paradox and the type semigroup

In this article we will study a key concept in relation to the well-known Banach-Tarski paradox. This is the concept of equidecomposability of subsets of a set X with respect to the action of a discrete group G . A subset E of X is G -paradoxical if there are two disjoint subsets E_1 and E_2 of E such that each of them is equidecomposable with E . The study of this relationship can be systematized by introducing a specific semigroup $S(X, G)$, called the *type semigroup* of X . We will explain the use of the type semigroup $S(X, G)$ in the proof of Tarski's Theorem. We will also introduce some generalizations of this concept to a topological setting, and we will consider the problem of the validity of Tarski's Theorem within this new context. In addition, we will review a recent result by Grabowski, Máthé and Pikhurko which gives a positive solution to the measurable circle squaring problem.

Keywords: Banach-Tarski paradox, type semigroup, Tarski's Theorem, topological space, circle squaring.

MSC2010 Subject Classification: 43A07, 03E25, 20E05.

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Compactness, a seminal idea in the evolution of general topology (1895–1930)

In this paper we analyze the evolution of the notion of compactness for topological spaces from its first appearance in a Fréchet 1904 paper generalising the Bolzano-Weierstrass property, encompassing its relationship with the Borel covering theorem for the interval $[0, 1]$, and including the general definition based on the Heine-Borel property for arbitrary open covers in 1929. We focus our analysis on four key works: Borel's thesis, Fréchet's thesis, Hausdorff's *Grundzüge* and Alexandroff-Urysohn's memoir.

Keywords: complete accumulation point, open covering, compact space.

MSC2010 Subject Classification: 01-02, 54-03.

Oriol Serra

Graph limits

The theory of graph limits was developed in the 2000s with the aim of creating a mathematical model of large networks. The need for such a model was mainly motivated by the emergence of the Internet with its huge network of interconnections, and large networks are also present in many scientific and technological areas, from biology, physics or social sciences to computer science, among others. László Lovász has been the main leader of the creative process behind this theory, and this was one of the reasons he was awarded the Hypatia European Prize in 2019 by the Barcelona Knowledge Hub of the Academia Europaea. In this paper, the basic notions and results of the theory of graph limits are presented and we offer a perspective on its main current developments.

Keywords: large networks, complex networks, graph theory, extremal combinatorics.

MSC2010 Subject Classification: 05C35, 05C25, 05C80, 05C82, 90B15.
