

## English summaries

**Esther Barrabés and Mercè Ollé**

*Space manifold dynamics: the highways of the universe*

In this article we seek to illustrate how the understanding of the dynamics of some models of Celestial Mechanics allows both to explain some astronomical phenomena and to design realistic space missions. The paradigmatic model used is the restricted three-body problem in which the invariant manifolds of the so-called libration orbits, that is, periodic and quasi-periodic orbits around the collinear equilibrium points of the model, play a key role. We will describe some of these phenomena and mention a number of specific missions. Lastly, we will comment on other useful (and more sophisticated) models in Astrodynamics and we will conclude with a remark on how the tools of Dynamical Systems can be transferred from the macroscopic (celestial) world to microscopic ones, such as classic atomic physics.

**Keywords:** restricted problem, invariant manifolds, equilibrium points, periodic and quasi-periodic orbits, space missions, homoclinic and heteroclinic connections.

**MSC2010 Subject Classification:** 70F07, 70F10, 70F15, 70H12, 70H33, 70K44.

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**Ramsès Fernández-València**

*Isogenies, codes and lattices in post-quantum cryptography*

In this paper we give a brief introduction to some of the concepts and mathematical techniques that are being explored in post-quantum cryptography. We introduce lattice theory together with code theory and the theory of isogenies of supersingular elliptic curves.

**Keywords:** lattice, code, isogeny, supersingular elliptic curve.

**MSC2010 Subject Classification:** 11T71, 14G50, 94A60.

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**Jordi Marzo***Minimal energy points and sphere packing*

In this article we deal with two very interesting problems and a way to relate them. The first problem is the study of the asymptotic development of the minimum energy of a set of points confined to a sphere and interacting through a Riesz potential. The limiting case of one of the constants that appear in this development will lead us to our second problem, that of determining the best sphere packing in the Euclidean space, a problem in which important advances have recently come out.

**Keywords:** Riesz energy, well-distributed points, sphere packing, linear programming.

**MSC2010 Subject Classification:** 31C20, 52C07, 11K36.

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**Lluís Vena***On two characterizations of Gauss codes*

In this paper we give some historical background, the motivation and some solutions to the *Gauss codes* problem.

A closed curve on the plane with  $n$  self-intersection points that are non-tangential, and where the curve goes through all points of the plane twice at the most, generates a double occurrence word on  $n$  symbols as follows. Label the intersection points with  $\{1, \dots, n\}$ , choose an initial point on the curve, a direction of travel through the curve, and record the sequence of self-intersection points found. Such word is said to be a *Gauss code* representable on the plane.

The Gauss codes problem seeks to characterize those double occurrence words that are representable by plane curves. We explain two different characterizations that involve 'turning the corner' to undo the crossing points in the two most natural ways. The first one leads to the characterization known on account of Dehn, and Read and Rosenstiehl. We present an alternative characterization that uses the other way of 'undoing' the crossing points and involves the Seifert cycles of the word. We also give some historical insight.

**Keywords:** Gauss codes, Gauss paragraph, alternative characterization, Seifert cycles.

**MSC2010 Subject Classification:** Primary: 05C10; Secondary: 57M15, 57M25.

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